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DELAY ANALYSIS FOR MULTIDIMENSIONAL QUEUEING PROCESS IN CSMA/CD LOCAL AREA NETWORKS

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A CSMA/CD local area network consists of single server (the channel) and multiple interacting queues of message packets. The message queueing process in a buffered, P-persistent CSMA/CD system is modeled as a multidimensional semi-markov chain. An effective approximation method to compute the mean packet delay in equilibrium is developed, based on the joint probability generating function of the queue length vector at embedded Markov epochs. We also develop a simulation model to validate approximation results. To the best of our knowledge, this work is the first in the literature that enables optimization of the control parameter P for the CSMA/CD system with more than two users.

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1. Introduction

Carrier Sense Multiple Access with Collision Detection (CSMA/CD) is a channel access protocol for packet broadcasting local area networks. This protocol allows user stations to contend for time on a shared channel by multiaccessing it in a random fashion. In an attempt to reduce the frequency with which broadcast message packets collide on the shared channel, Ethernet (Metcalfe and Boggs 1976) and IEEE 802 (IEEE 1985) use a technique known as 1-persistent CSMA/CD with *binary exponential backoff* (Goodman et al. 1988). An alternative technique proposed for reducing the frequency of collisions is *p*-persistent CSMA/CD (see, for example, Stallings 1987 and Takagi and Kleinrock 1985a); this protocol is considered in this paper.

The dynamic behavior and performance of *unbuffered* CSMA/CD systems has been studied extensively (e.g., Lam 1980, Takagi and Kleinrock 1985a, and Tobagi 1980). These studies assumed that there are an infinite number of network users and each user can have at most one packet in the message queue at any time (called an *infinite source model*). This simplifying assumption has been relaxed in recent studies on CSMA/CD systems: Apostolopoulos and Protonotarios (1986), Goodman et al. (1988), Hastad, Leighton and Rogoff (1987), Park and Bartoszyński (1990a), Park and Bartoszyński (1990b), Takagi and Kleinrock (1985b), and Tasaka (1986). In these studies, a finite number of users are considered to have message queues with finite or infinite capacity (called a *finite source model* or a *buffered* CSMA/CD system).

Among these studies on buffered CSMA/CD, the following authors considered *p*-persistent CSMA/CD. Under the assumption of finite capacity queues, Apostolopoulos and Protonotarios (1986) modeled the message queueing process as a two-dimensional semi-Markov chain. With the state space reduced to a two-dimensional space, it was necessary to employ an iterative approximation procedure to obtain the mean packet delay in steady state. The transmission probability p was arbitrarily set equal to the inverse of the number of busy users in the beginning of each contention period. Thus the dependency of the mean packet delay on the control parameter p was not investigated. Takagi and Kleinrock (1985b) developed a stationary joint probability generating function

(PGF hereinafter) of queue lengths for a system with *two* users, and obtained an explicit formula for the mean packet delay based on that PGF. An *exact* joint PGF of the queue length vector for a system with more than two users was first developed by Park and Bartoszyński (1990b). Further, they obtained stability conditions analytically based on the PGF.

In implementing p-persistent CSMA/CD the major problem is to determine the value of p so as to minimize the response time for users while maintaining stability of the system. In this paper an approximation procedure is developed to compute the optimal p which leads to a stationary operating mode and at the same time yields the minimum mean packet delay in equilibrium. An optimization of the p-value with respect to the response time has not been studied for a p-persistent CSMA/CD system with more than two users, though it has been carried out for slotted ALOHA by Saadawi and Ephremides (1981) and Sidi and Segall (1983).

In the next section, the p-persistent CSMA/CD channel access protocol is described and the queueing dynamics under this protocol is modeled as a multi-dimensional semi-Markov chain. A stationary PGF of packet backlog, i.e., the sum of all queue lengths, is obtained in Section 3 based on the joint PGF of the queue length vector. In Section 4 the Kolmogrov forward equation is derived applying an M/G/1 approximation to the stationary PGF of packet backlog, and an iterative procedure is developed to evaluate the mean packet delay in equilibrium and optimize p with respect to the mean delay. A numeric analysis is conducted in Section 5 and the analytic results are compared with simulation results demonstrating the accuracy of the proposed approximation scheme. Section 6 contains concluding remarks.

2. A Semi-Markovian Model of the Message Queueing Process

A brief description of the p-persistent CSMA/CD channel access protocol, which is sufficient for the reader to follow the subsequent analysis, follows.

The system consists of a single server (the channel) and multiple, infinite-capacity queues of customers (message packets). The number of queues, denoted by m, can be as large as 200 in local area networks. The channel time is slotted with the slot size being the maximum propagation delay, and a slot size is chosen as the unit of time. Discrete time index $t=0,1,\ldots$ is used to denote slot boundaries.

If the channel is busy no users attempt transmission (due to *carrier sensing*) until ongoing transmissions are completed and the channel becomes empty again. Once the channel becomes empty, every *busy user* (i.e., a user with a nonempty queue) persistently attempts at every t to transmit a message packet from the top of its queue. In a transmission attempt a user samples a random number from the uniform distribution over $[0,1]$. If the number is smaller than p, it starts transmission of a packet; otherwise, it suppresses the start of transmission. Thus whenever the channel is empty all busy users contend to seize the channel with an equal chance. If only one among all busy users samples a random number smaller than p at time t, the user starts transmitting a packet at t and the packet is successfully transmitted. If two or more users start transmitting packets at the same t, packets collide, the collision is detected, and all collided transmissions are aborted before transmissions are completed.

It is assumed that in each slot at most one packet arrives at a queue (which is reminiscent of Poisson process assumption). The probability that a packet arrives at a queue during a slot is denoted by λ. The numbers of packets arriving in different slots at the same user are independent, and the numbers of packet arrivals at different users in the same or different slots are independent. This message input process (called a *geometric arrival process*) implies that queueing processes at different users are symmetric; viz., queue lengths at individual users are *exchangeable* random variables (Billingsley 1979).

Following Takagi and Kleinrock (1985a), the channel state dynamics are modeled as depicted in Figure 1 (note that users 3 to m-1 in the figure are assumed to be idle throughout the portion of time shown in the figure). The channel state alternates between *idle periods* and *busy periods*. During an idle period all users have empty queues, and during a busy period there is at least one busy user. With the assumed message input process, durations of idle periods are independently and geometrically distributed; hence the beginning of an idle period is a system regenerative point. A busy period is divided into a number of *sub(busy)periods*, each in turn consisting of a *transmission delay* followed by a *packet transmission period.*

Figure 1 about here

During a transmission delay no packets are transmitted, even though the channel is empty, since all busy users continually sample random numbers exceeding p. A transmission delay terminates at time t when one or more busy users sample random numbers smaller than p and start packet transmissions. The duration of a successful packet transmission is proportional to the length of the packet transmitted. Packet lengths may vary. However, for analytic convenience, we use the expected value of packet lengths and denote it by ℓ. The duration of an unsuccessful transmission, which is the time to detect and resolve a collision, is represented by ℓ'. In a baseband system like Ethernet, it takes at most two slots to detect a collision. Thus ℓ' is usually much shorter than ℓ. A p-persistent CSMA/CD system is completely described by the set of parameters, $\{m,p,\lambda,\ell,\ell'\}$, at least for the purpose of our analysis.

Throughout the paper we use the following convention in defining notations: A boldface capital letter indicates an m-vector or a function with vector arguments, except that E and P stand for expectation and probability, respectively. An italic capital letter with an underlined superscript, $\underline{i}$, indicates the ith element of an m-vector, and an underlined italic capital letter signifies the sum of all elements in an m-vector. An italic capital letter without a superscript or an underscore signifies a set or a function with scalar arguments.

Denoting the set of network users by $I=\{1,2,..,m\}$, we define $\mathbf{Q}_t= [Q_t^{\underline{i}}, i\in I]$ to be the vector of queue lengths at t, and $\underline{Q}_t= \Sigma_{i=1}^{m} Q_t^{\underline{i}}$ to be the *packet backlog* at t. The process $\{\mathbf{Q}_t\}$ is characterized as an irreducible, aperiodic discrete-time Markov chain whose state space is the m-dimensional vector space $\Omega=\{[\omega_i,\ i\in I]:\ \omega_i\geq 0,\ \text{integer}\}$.

A semi-Markov chain model of the queueing process is constructed as follows (which is more accurate than the semi-Markov chain model presented in Takagi and Kleinrock 1985b): The beginning and the end of each idle period are regarded as embedded Markov epochs. Within a busy period, the boundaries of subperiods are chosen as embedded Markov epochs. Denoting these epochs by $k=0,1,\ldots$, with $k=0$ at $t=0$, we have an embedded Markov chain $\{\mathrm{Q}_k\}$ which is a multidimensional random walk. In Figure 1, the embedded epochs are shown by •, and the packet backlog at those epochs, Q_k, are illustrated.

3. Stationary Probability Generating Function of Packet Backlog

The major difficulty in analyzing the queueing process $\{\mathrm{Q}_k\}$ lies in that queues at different users interfere with each other through the shared channel. This problem of *interacting multiple queues* cannot be solved using classical queueing theory. There are no analytic results in the literature for evaluating the mean packet delay in such a multidimensional queueing system, and even approximations are difficult to obtain when $m>2$ (see Kleinrock and Yemini 1980). In this paper we base the derivation of the mean packet delay in equilibrium on the stationary PGF of $\{Q_k\}$. By the aggregation of $\{\mathrm{Q}_k\}$ into a one-dimensional process $\{Q_k\}$ we, of course, lose information about behaviors of individual queues. The approximation needed to overcome this loss of information can, however, be localized in the delay analysis (as will be shown in Section 4) so as to generate accurate results (verified by simulation in Section 5).

To describe the process $\{Q_k\}$ we first define the following random variables. Notations for each element in a random vector and the sum of all elements in a random vector are defined according to the convention mentioned earlier.

$\mathrm{X}(n)$: numbers of packet arrivals at respective queues over an n-slot period.

Z_k : numbers of packets that are successfully transmitted from respective queues at the end of the subperiod which begins at the kth epoch; Note that either $Z_k=0$ (0-vector) or $Z_k^i=1$ for some i and $Z_k^j=0$ for all $j\neq i$.

R_k : duration (in number of slots) of the transmission delay in the subperiod beginning at the kth epoch.

L_k : duration (in number of slots) of the packet transmission period in the subperiod beginning at the kth epoch.

U_k $:= Q_{k+1} - Q_k$.

B_k $:= \{i\in I:\ Q_k^i \geq 1\}$, i.e., the set of busy users at the kth epoch.

$B_k'(R_k) := \{i\in I:\ Q_k^i=0, X^i(R_k)\geq 1\}$, i.e., the set of users who are idle at the kth epoch, but become busy by the end of the transmission delay in the subperiod beginning at the kth epoch.

The process $\{\underline{Q}_k\}$ can now be recursively defined as

$$\underline{Q}_{k+1} = \underline{Q}_k + \underline{U}_k \quad \text{where } \underline{U}_k = \hat{\underline{X}}(1) \text{ if } \underline{Q}_k=0;\ \underline{U}_k = \underline{X}(R_k+L_k) - \underline{Z}_k \text{ otherwise.} \qquad (1)$$

In this equation $\hat{\underline{X}}(1)$ represents random variable $\underline{X}(1)$ with the constraint $\underline{X}(1) \geq 1$. Under the geometric input process, $\underline{X}(1)$ has a binomial distribution with parameters m and λ. Therefore, if $\underline{Q}_k=0$ (i.e., if the kth epoch is the beginning of an idle period), $\underline{Q}_{k+1}$ follows a truncated binomial distribution given by

$$P\{\underline{Q}_{k+1}=q \mid \underline{Q}_k=0\} = \binom{m}{q}\lambda^q(1-\lambda)^{m-q} \Big/ \{1-(1-\lambda)^m\}, \quad 1\leq q\leq m. \qquad (2)$$

Note that $\underline{Q}_{k+1}$ given $\underline{Q}_k=0$ is the packet backlog at the start of a busy period.

Next, if $\underline{Q}_k \geq 1$, the random drift $\underline{U}_k$ is the sum of random variables $\underline{X}(R_k)$, $\underline{X}(L_k)$ and $-\underline{Z}_k$. Unfortunately, the latter variables are stochastically dependent on each other in a complex way. First, L_k (and hence $\underline{X}(L_k)$) depends on $\underline{Z}_k$. Namely, $L_k=\ell'$ if $\underline{Z}_k=0$ and $L_k=\ell$ otherwise (i.e., if $\underline{Z}_k=1$). Random variable $\underline{Z}_k$ in turn depends on $|B_k|$ and $|B_k'(R_k)|$ since the success of a packet transmission depends on the number of busy users at the end of the transmission delay, which is the sum of $|B_k|$ and $|B_k'(R_k)|$. The latter, $|B_k'(R_k)|$, depends on $\underline{X}(R_k)$. Further, R_k and $\underline{X}(R_k)$ are jointly dependent on $|B_k|$, and finally $|B_k|$ is uniquely determined given Q_k, but stochastically dependent on $\underline{Q}_k$. These stochastic dependencies are summarized in Lemma 1 below. A detailed analysis of these stochastic dependencies and the resultant conditional PGF of Q_{k+1} given $Q_k \geq 1$ can be found in Park and Bartoszyński (1990b). A derivation of the latter PGF and the proof of Lemma 1 are summarized in Appendix I at the end of this paper.

<u>Lemma 1</u>: *For any k with* $\underline{Q}_k>0$,

$$E(\underline{U}_k|\underline{Q}_k)= EE\left\{\underline{X}(R_k)+EE\left[E\left[\underline{X}(L_k)\middle|\underline{Z}_k\right]-\underline{Z}_k\middle| |B_k'(R_k)| \middle| R_k,\underline{X}(R_k)\right]\middle| |B_k| \middle| \underline{Q}_k\right\}.$$

An optimization of the p-value with respect to the mean packet delay has been studied for slotted ALOHA, as mentioned earlier in Section 1. Compared with slotted ALOHA, the delay analysis and optimization of the p-value is significantly more complex for p-persistent CSMA/CD. The source of additional complexity is the stochastic dependencies among the random variables as described in Lemma 1. These stochastic dependencies stem from the *carrier sensing* and *collision detection* mechanisms which have been added to the slotted ALOHA protocol in devising p-persistent CSMA/CD (see Szpankowski 1988 and Tsybakov and Mikhailov 1980 for a Markov chain model of slotted ALOHA with buffered users). In this paper the stochastic dependencies among L_k, $\underline{Z}_k$, R_k, $\underline{X}(R_k)$ and $|B_k|$ are *exactly* incorporated in the delay analysis. An approximation is introduced to simplify the dependency of $|B_k|$ on $\underline{Q}_k$.

The process $\{\underline{Q}_k\}$ is a random walk with random drifts $\{\underline{U}_k\}$. The process is *uniformly downward bounded* meaning that $P(\underline{U}_k<-1)=0$ for all k. It also possesses an important property called the *bounded homogeneity*, which is described in the following Lemma (see Appendix I for the proof):

<u>Lemma 2</u>: *For any* k, k', $\underline{Q}_k$ *and* $\underline{Q}_{k'}$ *such that* $|B_k|=|B_{k'}|$, $P(\underline{U}_k=u|\underline{Q}_k)= P(\underline{U}_{k'}=u|\underline{Q}_{k'})$.

Define $F_k(s)$ to be the PGF of $\underline{Q}_k$. In view of (1) we write

$$F_{k+1}(s)= F_{k+1}(s|Q_k=0)P(Q_k=0) + F_{k+1}(s|Q_k>0)P(Q_k>0). \tag{3}$$

Letting $H(s):= F_{k+1}(s|Q_k=0)$, we obtain from Equation (2)

$$H(s)= \{(\lambda s+1-\lambda)^m-(1-\lambda)^m\}/\{1-(1-\lambda)^m\}. \tag{4}$$

By Lemmata 1 and 2 we may write

$$F_{k+1}(s|Q_k>0)= \mathrm{E}\left[s^{Q_k}\mathrm{EE}\left\{s^{\underline{U}_k}\middle| |B_k|\right\}\middle| Q_k>0\right]=$$

$$= \frac{1}{\mathrm{P}(Q_k>0)} \sum_{q=1}^{\infty} s^q \sum_{b=1}^{\min(q,m)} \sum_{u=-1}^{\infty} s^u\ \mathrm{P}(\underline{U}_k=u|\,|B_k|=b)\ \mathrm{P}(|B_k|=b|Q_k=q)\ \mathrm{P}(Q_k=q). \tag{5}$$

Define $G_k(s|\,|B_k|=b):= \Sigma_{u=-1}^{\infty}s^u\mathrm{P}(\underline{U}_k=u|\,|B_k|=b)$, i.e., the conditional PGF of $\underline{U}_k$ given $|B_k|=b$. By Lemma 2, for any $b\neq0$, $G(s|b):= G_k(s|\,|B_k|=b)$ is independent of k and Q_k. By setting $s^{\underline{i}} =s$ for all $i\in I$ in Equation (A4) in Appendix I, we get

$$G(s|b)= \sum_{j=0}^{m-b} \binom{m-b}{j} \left(\frac{\lambda s}{(1-p)(\lambda s+1-\lambda)-(1-\lambda)}\right)^j \times$$

$$\times \left[s^{-1}(\lambda s+1-\lambda)^{m\ell}(b+j)p(1-p)^{b+j-1} +(\lambda s+1-\lambda)^{m\ell'}\{1-(1-p)^{b+j}-(b+j)p(1-p)^{b+j-1}\}\right] \times$$

$$\times \left\{\sum_{n=0}^{j} \binom{j}{n}(-1)^{j-n}\left[1-(1-p)^{b+n}(1-\lambda)^{m-b-n}(\lambda s+1-\lambda)^{b+n}\right]^{-1}\right\}. \tag{6}$$

Note that index j in Equation (6) represents the number $|B'_k(R_k)|$.

By substituting (6) into (5), (4)-(5) into (3), and then by passing k to ∞ in (3), the stationary PGF of $\{Q_k\}$, denoted by $F(s)$, is written as

$$F(s) = H(s)P(Q=0) + \sum_{q=1}^{\infty} s^q \sum_{b=1}^{\min(q,m)} G(s|b)P(|B|=b|Q=q)P(Q=q). \qquad (7)$$

As in (7) we simply drop the time index k from an argument to indicate its quantity in steady state.

4. Delay Analysis

In the sequel, we write $F^{(j)}(c)$ to denote the jth partial derivative of $F(s)$ with respect to s, evaluated at $s=c$. We need to evaluate $F^{(1)}(1)$ to obtain the mean packet backlog at an embedded epoch in steady state, which in turn is used to evaluate the mean packet delay. Analytic evaluations of $F(s)$ and its derivatives are difficult to obtain mainly because the conditional probability $P(|B|=b|Q=q)$ cannot be evaluated explicitly in the presence of interactions among queues. Therefore, we introduce an M/G/1 approximation in formulating this probability.

In this approximation, we assume that in steady state the message queue at a user behaves like a discrete M/G/1 queue *independently* of other queues. The queue at a user in the CSMA/CD system indeed has a single server (the channel) and geometric arrivals (a discrete version of Poisson arrivals) of message packets. But how about the service time? The service time for a packet begins at the point when the packet is promoted to the top of its queue. The service time is defined, in the context of an M/G/1 system, to consist of: (i) a number of sub(busy)periods, since that beginning point, during each of which the packet is not transmitted (due to sampling a random number greater than p) or is involved in a collision; plus (ii) the subperiod in which the packet is finally transmitted with success. This service time is actually affected by other busy users' contention for the server. Furthermore, the set of busy users may change during the service time of a particular packet. The assumption of independence (or "no interference") among the queues in steady state is thus introduced so that service times are i.i.d. in steady state.

With this assumption, we now may write

$$\mathrm{P}(|B|=b)= \binom{m}{b}\theta^{b}(1-\theta)^{m-b}, \tag{8}$$

where θ is defined to be the steady-state probability of a user having a non-empty queue at an embedded epoch.

The *no interference* assumption for the steady-state behavior of multiple queues should lead to a closer approximation when it is applied to *p*-persistent CSMA/CD than when applied to slotted ALOHA. This is because the former suffers less from interference (viz., collisions) than the latter due to additional controls, *carrier sensing* and *collision detection.* The approximation in (8) is motivated by the results in Sidi and Segall (1983) where an M/M/1 approximation is applied to the slotted ALOHA demonstrating good accuracy over a wide range of system parameters. It should be noted that the approximation is limited only to computing θ, and the rest of delay analysis exactly reflects the dynamic behavior of interacting multiple queues.

Incorporating (8) into (7) and using the equation $F(0)= \mathrm{P}\{Q=0\}$, we obtain

$$F(s|\theta)= H(s)F(0|\theta) + V(s|\theta)\{F(s|\theta)-F(0|\theta)\}, \tag{9}$$

where $$V(s|\theta):= \sum_{b=1}^{m} G(s|b)\binom{m}{b}\theta^{b}(1-\theta)^{m-b}. \tag{10}$$

Taking the first derivatives of both sides of (9) yields

$$F^{(1)}(s|\theta)= \left[H^{(1)}(s)F(0|\theta) + V^{(1)}(s|\theta)\{F(s|\theta)-F(0|\theta)\}\right] \Big/ \{1-V(s|\theta)\}. \tag{11}$$

By evaluating Equation (11) at s=1 and using the condition $F(1|\theta)= V(1|\theta)= 1$, the equilibrium differential equation (i.e., Kolmogrov forward equation in equilibrium: see Kleinrock 1975) can be derived as

$$0 = H^{(1)}(1)F(0|\theta) + V^{(1)}(1|\theta)\{1-F(0|\theta)\}, \tag{12}$$

which can be rewritten as

$$F(0|\theta) = V^{(1)}(1|\theta)/\{V^{(1)}(1|\theta)-H^{(1)}(1)\}. \tag{13}$$

Here $H^{(1)}(1)$ and $V^{(1)}(1|\theta)$ are the expected drift of packet backlog during an idle period and that during a sub(busy)period, respectively. From (4) we get $H^{(1)}(1) = m\lambda/\{1-(1-\lambda)^m\}$. $V^{(1)}(1|\theta)$ is obtained simply by replacing $G(s|b)$ by $G^{(1)}(1|b)$ in the RHS of Equation (10). $G^{(1)}(1|b)$ is the expected drift of packet backlog over a subperiod beginning with b busy users, and is derived in Appendix II using Equation (6).

In light of (12), the condition $V^{(1)}(1|\theta)< 0$ must be satisfied for the system to achieve equilibrium. In Equation (10) we see that $V^{(1)}(1|\theta)$ is the marginal expectation of $G^{(1)}(1|b)$ over $b=1,..,m$, which in turn is equivalent to $E(\underline{U}_k|\underline{Q}_k>0)$. Thus the equilibrium equation (12) implies that a necessary condition for stability of p-persistent CSMA/CD (i.e., *ergodicity* of $\{Q_t\}$) is *approximately* given by $E(\underline{U}_k|\underline{Q}_k>0)< 0$. Furthermore, by Pakes' (1969) lemma, the latter condition is also sufficient for ergodicity of the one-dimensional random walk $\{\underline{Q}_k\}$. It can be easily proved that $\{Q_t\}$ is ergodic if and only if $\{\underline{Q}_k\}$ is ergodic. Therefore, the condition $E(\underline{U}_k|\underline{Q}_k>0)< 0$ can be regarded as a sufficient and necessary condition for stability. However, a theoretical proof of this assertion is difficult and has not been established in the literature.

The reader is referred to Park and Bartoszyński (1990b) for a theoretical analysis of stability conditions for p-persistent CSMA/CD. Some of the main results in that paper are summarized in the following for later reference in Section 5. The conjecture below has been numerically established.

<u>Lemma 3</u>: $\{Q_t\}$ *is recurrent if* $G^{(1)}(1|b)=E(\underline{U}_k||B_k|=b) \le 0$ *for all* $b=1,..,m$, *and only if* $m\lambda[\ell+(1-p)^m/\{1-(1-p)^m\}] \le 1$.

<u>Conjecture</u>: *Given system parameters* $\{m,\lambda,\ell,\ell'\}$, *for any* b *there exist* $p_L(b)$ *and* $p_U(b)$ *such that* $G^{(1)}(1|b)\le 0$ *if and only if* $p_L(b)\le p\le p_U(b)$.

Further, $\{Q_t\}$ *is recurrent if* $p_L(1) \le p \le p_U(m)$ *since* $p_L(b) < p_L(b')$ *and* $p_U(b) < p_U(b')$ *for any* $b > b'$.

Recall that θ is the probability of a user being busy in steady state, and that $F(0|\theta)$ is the conditional probability of all users being idle in steady state given the value of θ. It follows that, under the assumption of independent queues, we must have $1-\theta = F(0|\theta)^{1/m}$. We thus compute the value of θ iteratively using Equation (13) until the following convergence criterion is met:

$$|\theta - \{1-F(0|\theta)^{1/m}\}| < \epsilon. \tag{14}$$

The function $f(\theta) := \theta + F(0|\theta)^{1/m} - 1$ is monotone increasing in θ and changes sign once in the range $(0,1)$ of θ. Therefore, a bisecting search method can quickly find the value of θ satisfying (14) even for a extremely small value of ϵ.

Let $\hat{\theta}$ be the value of θ satisfying (14). We can now evaluate $F^{(1)}(1|\hat{\theta})$ by applying L'Hospital's rule to (11) and then replacing the RHS of Equation (13) for $F(0|\hat{\theta})$:

$$F^{(1)}(1|\hat{\theta}) = \left[H^{(1)}(1)\ V^{(2)}(1|\hat{\theta}) - H^{(2)}(1)\ V^{(1)}(1|\hat{\theta})\right] \Big/ \; 2\ V^{(1)}(1|\hat{\theta}) \left[V^{(1)}(1|\hat{\theta}) - H^{(1)}(1)\right]. \tag{15}$$

In this equation, $H^{(2)}(1) = m(m-1)\lambda^2/\{1-(1-\lambda)^m\}$ from (4), and $V^{(2)}(1|\hat{\theta})$ is obtained simply by replacing $G(s|b)$ by $G^{(2)}(1|b)$ in the RHS of Equation (10). A derivation of $G^{(2)}(1|b)$ is provided in Appendix II.

Recall that $F^{(1)}(1|\hat{\theta})$ is the mean packet backlog observed only at the embedded Markov epoch. In order to obtain the steady-state mean packet delay observed at an arbitrary slot boundary t, we need to evaluate the mean packet backlog at an arbitrary t in steady state (see Takagi and Kleinrock 1985b for a similar treatment). If t is in an idle period, the backlog is zero. The probability that an arbitrary t is found in a busy period when the system is in steady state is given by $\{1-F(0|\hat{\theta})\}$. Letting $\bar{Q}$ denote the mean packet backlog at an arbitrary t in steady state, we have

$$\bar{Q}= (1-F(0|\vartheta))S/T \tag{16}$$

where T is the expected duration of a sub(busy)period in steady state, and S is the expected backlog accumulation over a subperiod in steady state.

First, to evaluate T, we note that the duration of a subperiod is i.i.d. for all subperiods starting with the same number of busy users, due to the *bounded homogeneity* (see Lemma 2). Letting $\delta(b)$ denote the expected duration of a subperiod starting with b busy users, we may write $T= \Sigma_{b=1}^{m}\delta(b)\mathrm{P}(|B|=b)$. Furthermore, due to *regeneracy* of the queueing process the duration of a subperiod starting with b (>0) busy users converges a.s. to $\mathrm{E}(R_1+L_1||B_1|=b)$ provided that the process is ergodic. Consequently, for any q>0 we have

$$\delta(b)= \mathrm{E}(R_1+L_1||B_1|=b)= \mathrm{E}\left\{R_1+\mathrm{E}\left[\mathrm{E}\left[L_1\middle|\underline{Z}_1\right]\middle|R_1,|B_1'(R_1)|\right]\middle||B_1|=b\right\}, \tag{17}$$

reflecting the stochastic dependencies among R_k, L_k and $\underline{Z}_k$ (see Lemma 1).

Define $\Gamma(s|b)$ to be the conditional PGF of (R_1+L_1) given $|B_1|=b>0$. This PGF and $\delta(b)= \Gamma^{(1)}(1|b)$ are derived based on (17) in Appendix III. Using (A9) in the Appendix and Equation (8) we get

$$T= \sum_{b=1}^{m}\Gamma^{(1)}(1|b)\binom{m}{b}\vartheta^b(1-\vartheta)^{m-b}. \tag{18}$$

Turning to S in (16), let us first define $\beta(q)$ as the expected backlog accumulation over a subperiod beginning with q outstanding packets so that $S= \Sigma_{q=1}^{\infty}\beta(q)\mathrm{P}(Q=q)$. To derive $\beta(q)$ for q>0, we define $\xi(y|q,b)$ to be the total time (in number of slots) that $\{Q_t\}$ spends in state y during the subperiod which starts with q packets waiting at b (≥1) busy users. Due to *bounded homogeneity*, $\xi(y|q,b)$ is i.i.d. for all subperiods that start with q waiting packets and b busy users. Further, it converges a.s. to its expected value if the queueing process is ergodic. Therefore, for any q>0 we have

$$\beta(q)=\sum_{y=q}^{\infty}\sum_{b=1}^{\min(q,m)} y\,\mathrm{E}\xi(y|q,b)\;\mathrm{P}(|B|=b|Q=q). \tag{19}$$

The process $\{Q_t\}$ is non-decreasing during a subperiod and the packet arrival process is time homogeneous. Consequently, the average backlog at an arbitrary slot boundary in the subperiod which starts with q waiting packets and b busy users, or $\Sigma_{y=q}^{\infty} y\mathrm{E}\xi(y|q,b)/\delta(b)$, is estimated by $q+(.5)m\lambda\delta(b)$. We thus can write

$$S=\sum_{q=1}^{\infty}\beta(q)\mathrm{P}(Q=q)=\sum_{q=1}^{\infty}\sum_{b=1}^{\min(q,m)}\delta(b)\{q+(.5)m\lambda\delta(b)\}\;\mathrm{P}(|B|=b|Q=q)\mathrm{P}(Q=q). \tag{20}$$

From (9) we see that $\Sigma_{q=1}^{\infty} q\mathrm{P}(Q=q)= d[V(s|\theta)\{F(s|\theta)-F(0|\theta)\}]/ds|_{s=1}= F^{(1)}(1|\theta)+V^{(1)}(1|\theta)\{1-F(0|\theta)\}$. Incorporating the latter and (8) into (20) we get

$$S= T\left[F^{(1)}(1|\hat{\theta})+V^{(1)}(1|\hat{\theta})\{1-F(0|\hat{\theta})\}\right]+(.5)m\lambda\sum_{b=1}^{m}\Gamma^{(1)}(1|b)^2\binom{m}{b}\hat{\theta}^b(1-\hat{\theta})^{m-b}. \tag{21}$$

Finally, using Little (1961)'s theorem the mean packet delay in steady state, D, is given by

$$D=\bar{Q}/m\lambda=\{1-F(0|\hat{\theta})\}S/m\lambda T. \tag{22}$$

The mean packet delay, D, includes (i) the elapsed time from the arrival of a packet until the packet is promoted to the top of the queue, (ii) a number of sub(busy)periods during each of which the packet is not transmitted or is involved in a collision, and (iii) the subperiod in which the packet is successfully transmitted.

5. Numeric and Simulation Experiments

The algorithm to compute D given parameters $\{p,m,\lambda,\ell,\ell'\}$ was encoded in Fortran 77; it first finds $\hat{\theta}$ and $F(0|\hat{\theta})$ satisfying the condition (14) with ϵ=1.E-10, and then compute D using Equations (15) through (22).

Computational results are shown in Tables 1 and 2, respectively, for two different cases. The two cases have the same values for m (=50) and ℓ' (=3), but different values for λ and ℓ. The latter parameters were set to 1.6E-4 and 75, respectively, in Case 1 to represent a high utilization of the channel capacity; and to 1.2E-4 and 25, respectively, in Case 2 to represent a low utilization. Note that by Lemma 3 an upper bound on the *channel capacity* (i.e., the maximum achievable throughput rate in equilibrium) can be given by $1/\ell$. The ratio of the total input rate, $m\lambda$, to $1/\ell$ is 62% in Case 1 and 15% in Case 2.

Tables 1 and 2 about here

In view of the conjecture in Section 4, the range $[p_L(1), p_U(m)]$ of p that guarantees stability is provided in each table. Among the steady-state measures reported in the tables, the new notation, B^+, signifies the set $\{b: G^{(1)}(1|b) > 0\}$. The p-values, at which $V^{(1)}(1|\hat{\theta}) < 0$ and B^+ is nonempty, yield a finite mean packet delay, though not satisfying the sufficient condition for stability given in Lemma 3. It should be noted, however, that the delay at $p=p_U(m)$ is close to the minimum delay: in Case 1 the minimum delay was 128.6 at the optimal p-value .16, while the delay at $p=p_U(50)=.08$ was 140.4, exhibiting a 9% gap from the minimum; in Case 2 the gap was only 2%. It is also notable that the expected number of busy users in steady state, or $E(|B|)$ ($= m\hat{\theta}$), was 1.5 and 1.1, respectively, in Cases 1 and 2 when p was set at the optimum. This indicates that there is little interference among queues in steady state if p is optimized with respect to the mean packet delay, which justifies the M/G/1 approximation. Indeed, the ratio of collisions to all packet transmissions observed in simulation runs was about 5% and 1% in Cases 1 and 2, respectively, with the optimal p-values.

The simulation model was written in SLAM II (Pritsker 1984) and run on an IBM 3033AP mainframe. In the simulation model, we generated packet arrivals for each user so that the interarrival times follow an exponential distribution with intensity λ. It is well known that the geometric distribution of interarrival times can be closely approximated by the exponential distribution when λ is small (Feller 1968). This exponential approximation tremendously reduced the computing time. Approximately 5,000 packet transmissions were collected to estimate the mean packet delay for each p-value. The first 500 observations were discarded to eliminate the initial transient period (see Wilson and Pritsker 1978, Schruben and Goldsman 1985).

The simulation results are shown in Figure 2 along with the numerical approximation results. The solid line and dotted line show approximation results for Cases 1 and 2, respectively. Simulation results for Cases 1 and 2 are plotted using symbols '+' and '*', respectively, and show good agreement with approximation results. The delay (versus p) curve is U-shaped and the minimum delay is attained in the neighborhood of the maximum p-value that leads to equilibrium (viz., near the bottom right corner of the U curve). A similar, yet different, pattern of the delay curve was found for slotted ALOHA systems by Sidi and Segall (1983). The CSMA/CD achieves close-minimal mean packet delays over a much wider range of the p-value than the slotted ALOHA, presumably due to the additional medium access control mechanisms — carrier sensing and collision detection.

Figure 2 about here

6. Conclusion

A multidimensional queueing process with interactions among individual queues arises in many computer and communication systems such as coupled processors (Szpankowski 1988), ALOHA satellite communication (Sidi and Segall 1983) and CSMA/CD local area networks. However, there are no analytic results for computing the mean queueing delay in such a system with more than two queues.

In this paper we have shown that the mean packet delay in CSMA/CD networks can be obtained quite accurately using an M/G/1 approximation. The success of this approximation method is ascribed to the following: (i) The message queueing process has the bounded homogeneity property (Lemma 2). With this property, after introducing the approximate distribution of the number of busy users (Equation (8)), we could simplify the stationary PGF of the packet backlog (Equation (7)) into a product form (Equation (9)). (Note that in Equation (9), $\Sigma_{q=1}^{\infty} P(Q=q)$ in Equation (7) is factored out as $\{F(s|\theta)-F(0|\theta)\}$.) (ii) The delay analysis incorporates an exact PGF for the random drift of the packet backlog over a period between two consecutive embedded Markov epochs (Equations (2), (6) and Lemma 1). (iii) Finally, the interference among queues is actually minimal in steady state if the control parameter p is optimized.

A practical and easy way to determine the p-value would be to set it equal to $p_U(m)$; this guarantees stability (by Lemma 3 and Conjecture) and at the same time yields a close-minimal mean packet delay in equilibrium (as numerically demonstrated in Section 5). According to Conjecture, for given system parameters $\{m,\lambda,\ell,\ell'\}$, $p_U(m)$ is obtained by solving $G^{(1)}(1|m)=0$ for p.

Substituting m for b in Equation (A5) in Appendix II, we obtain

$$G^{(1)}(1|m)=\frac{m}{1-(1-p)^m}\left[\lambda(1-p)^m+(m\lambda\ell-1)p(1-p)^{m-1}+\lambda\ell'\{1-(1-p)^m-mp(1-p)^{m-1}\}\right]. \quad (23)$$

Rewriting (23) we see that $G^{(1)}(1|m)=0$ is equivalent to

$$f(q):=-c_1q^m+c_2q^{m-1}+c_3=0, \quad (24)$$

where $q=1-p$, $c_1=c_2+\lambda(\ell'-1)$, $c_2=m\lambda\{\ell-\ell'\}-1$ and $c_3=\lambda\ell'$. The equation $f(q)$ has the global minimum at $q^*=(m-1)c_2/mc_1$. It has exactly two solutions (q_1,q_2) such that $0<q_1<q^*<q_2<1$ provided that $m\lambda\ell\leq 1$ (which is a necessary condition for stability by Lemma 3) and $f(q^*)<0$. A bisecting search over the range $(0,q^*)$ of q can quickly find q_1, and then we have $p_U(m)=1-q_1$.

Our approach -- viz., incorporating an exact transient analysis of multiple interacting queues into an M/G/1 approximation for evaluating average delay in steady state -- can be employed to analyze other CSMA/CD protocols. It would be interesting to attempt applying this approach to the analysis of new variants of CSMA/CD that are designed for high-speed broadcast bus networks (e.g., p_i*-persistent protocol* by Mukherjee and Meditch (1988), *LCSMA-CD* by Maxemchuk (1988), and a modified CSMA/CD by Lin and Sousa (1990)).

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Appendix I. The Conditional Joint PGF of Q_{k+1} Given $Q_k \neq 0$

The joint probability of $R_k = r$ and $X(r) = [x_i, i \in I\colon 0 \le x_i \le r]$ conditional on Q_k is given by

$$P(R_k = r, X(r) = [x_i, i \in I] \mid Q_k) = \left\{1-(1-p)^{|B_k| + |B_k'(r)|}\right\}\left\{\prod_{i \in B_k} (1-p)^r \binom{r}{x_i} \lambda^{x_i} (1-\lambda)^{r-x_i}\right\} \times$$

$$\left\{\prod_{i \in I - B_k - B_k'(r)} (1-\lambda)^r\right\}\left\{\prod_{i \in B_k'(r)} \sum_{w=1}^{r} (1-\lambda)^{w-1} \lambda (1-p)^{r-w} \binom{r-w}{x_i - 1} \lambda^{x_i - 1} (1-\lambda)^{r-w-x_i+1}\right\}. \quad \text{(A1)}$$

In this equation, w represents the slot in which the first packet since the beginning of the kth subperiod arrives at user i (who had no waiting packets in the beginning of the subperiod). It should be noted that $B_k'(r)$ is uniquely determined given B_k and $X(r)$, and B_k is uniquely determined given Q_k. Equation (A1) thus shows that the joint distribution of R_k and $X(R_k)$ depends only on B_k, apart from the system parameters p, λ, and I. That is, the joint distribution is identical for any k and k' with $B_k = B_{k'}$, even if $Q_k \neq Q_{k'}$.

The probability of having a collision in the subperiod beginning at the kth epoch, given $R_k = r$, is

$$P(Z_k = 0 \mid B_k, B_k'(r)) = \left\{1-(1-p)^{|B_k| + |B_k'(r)|} - (|B_k| + |B_k'(r)|)p(1-p)^{|B_k| + |B_k'(r)| - 1}\right\} /$$

$$\left\{(1-(1-p)^{|B_k| + |B_k'(r)|}\right\}. \quad \text{(A2)}$$

The probability of having a successful transmission in the subperiod beginning at the kth epoch, conditional on $R_k = r$, is simply given by $P(Z_k \neq 0 \mid B_k, B_k'(r)) = 1 - P(Z_k = 0 \mid B_k, B_k'(r))$. As such, given $R_k = r$, Z_k depends only on B_k and $B_k'(r)$. Recall that $B_k'(R_k)$ is uniquely determined by B_k and $X(R_k)$ and that R_k and $X(R_k)$ are jointly dependent on B_k. Therefore, Z_k is transitively dependent only on B_k.

Since $X(L_k)$ depends only on Z_k, it follows that the random drift U_k depends on Q_k only through B_k. We thus have proved both Lemmata 1 and 2.

Let $F_{k+1}(s|Q_k\neq 0)$, where $s=[s_i, i\in I]$, be the conditional PGF of Q_{k+1} given $Q_k\neq 0$, and $G(s|B) := G_k(s|B_k=B)$ be the conditional PGF of U_k given $B_k=B$ which is independent of k and Q_k (by Lemma 2). Then we have

$$F_{k+1}(s|Q_k\neq 0)= E\left(\prod_{i\in I} s_i^{Q_k^i} E\left\{\prod_{i\in I} s_i^{U_k^i}\middle| B_k\right\}\middle| Q_k\neq 0\right)=$$

$$= \frac{1}{P(Q_k\neq 0)}\sum_{b=1}^{m}\sum_{\forall B\subseteq I:|B|=b} G(s|B)\left(F_k(s:s_i=0\ \forall i\notin B) - \sum_{i'\in B} F_k(s:s_{i'}=0, s_i=0\ \forall i\notin B)\right). \quad \text{(A3)}$$

Using (A1) and (A2), function $G(s|B)$ is evaluated as:

$$G(s|B)= E\left\{\prod_{i\in I} s_i^{X^i(R_k)}\ E\left[\prod_{i\in I} s_i^{-Z_k^i}\ E\left[\prod_{i\in I} s_i^{X^i(L_k)}\middle| Z_k\right]\middle| R_k, X(R_k)\right]\ \middle| B_k=B\right\}=$$

$$=\sum_{r=0}^{\infty}\sum_{x_i=0\ \forall i\in I}^{r}\left[\prod_{i\in I} s_i^{x_i}\right]\sum_{\substack{z_i=0\ \forall i\in I\\ (z_1+\cdots+z_m\leq 1)}}^{1} E\left[\prod_{i\in I} s_i^{X^i(L_k)}\middle| Z_k=[z_i, i\in I]\right]\left[\prod_{i\in I} s_i^{-z_i}\right]\times$$

$$\times\ P\left(Z_k=[z_i, i\in I]\middle| R_k=r, X(r)=[x_i, i\in I], B_k=B\right)\ P\left(R_k=r, X(r)=[x_i, i\in I]\middle| B_k=B\right)=$$

$$=\sum_{j=0}^{m-b}\sum_{\forall B'\subseteq I-B:|B'|=j}\left\{\prod_{i\in B'} \lambda s_i\left[(1-p)(\lambda s_i+1-\lambda)-(1-\lambda)\right]^{-1}\right\}\times$$

$$\times\left\{\left(\Pi(s)p(1-p)^{b+j-1}\cdot\sum_{i\in B\cup B'} s_i^{-1}\right) + \Phi(s)\{1-(1-p)^{b+j}-(b+j)p(1-p)^{b+j-1}\}\right\}\times$$

$$\times\sum_{n=0}^{j}\sum_{\forall A\subseteq B':|A|=n}(-1)^{j-n}\left[1-(1-p)^{b+n}(1-\lambda)^{m-b-n}\prod_{i\in B\cup A}(\lambda s_i+1-\lambda)\right]^{-1} \quad \text{(A4)}$$

where $\Pi(s)=\prod_{i\in I}(\lambda s_i+1-\lambda)^{\ell}$ and $\Phi(s)=\prod_{i\in I}(\lambda s_i+1-\lambda)^{\ell'}$ are the PGF's of the duration of a successful and an unsuccessful transmission period, respectively. A detailed derivation of Equation (A4) can be found in Park and Bartoszyński (1990b).

Appendix II. Derivatives of the PGF $G(s|b)$ Evaluated at $s=1$

Taking the first and second derivatives of $G(s|b)$, which is given in Equation (6), with respect to s and evaluating the derivatives at $s=1$ we obtain

$$G^{(1)}(1|b)=\sum_{j=0}^{m-b}\binom{m-b}{j}\left(\frac{\lambda}{\lambda-p}\right)^{j}\Bigg(\psi\Big[\{1-(1-p)^{b+j}\}jp(1-\lambda)/(p-\lambda)+(\lambda m\ell-1)\alpha+$$

$$+\lambda m\ell'\{1-(1-p)^{b+j}-\alpha\}\Big]+\lambda\phi\{1-(1-p)^{b+j}\}\Bigg),\quad\text{and}\tag{A5}$$

$$G^{(2)}(1|b)=\sum_{j=0}^{m-b}\binom{m-b}{j}\left(\frac{\lambda}{\lambda-p}\right)^{j}\Bigg(\psi\Big[\alpha\{\lambda^2 m\ell(m\ell-1)-2\lambda m\ell+2\}+\lambda^2 m\ell'(m\ell'-1)\{1-(1-p)^{b+j}-\alpha\}\Big]+$$

$$+2\Big[(\lambda m\ell-1)\alpha+\lambda m\ell'\{1-(1-p)^{b+j}-\alpha\}\Big]\Big[\lambda\phi+jp(1-\lambda)\psi/(p-\lambda)\Big]+$$

$$+\{1-(1-p)^{b+j}\}\Big[\lambda^2(2\pi+\phi')+2jp\lambda(1-\lambda)\phi/(p-\lambda)\Big]+$$

$$+j\{1-(1-p)^{b+j}\}\psi\Big[j-1+2j\lambda(1-p)/(p-\lambda)+(j+1)\lambda^2(1-p)^2/(p-\lambda)^2\Big]\Bigg),\tag{A6}$$

where $\alpha:=(b+j)p(1-p)^{b+j-1}$; $\beta:=1-(1-p)^{b+n}(1-\lambda)^{m-b-n}$; $\psi:=\sum_{n=0}^{j}\binom{j}{n}(-1)^{j-n}/\beta$;

$\phi:=\sum_{n=0}^{j}\binom{j}{n}(-1)^{j-n}(b+n)(1-\beta)/\beta^2$; $\phi':=\sum_{n=0}^{j}\binom{j}{n}(-1)^{j-n}(b+n)(b+n-1)(1-\beta)/\beta^2$;

and $\pi := \sum_{n=0}^{j} \binom{j}{n} (-1)^{j-n} (b+n)^2 (1-\beta)^2/\beta^3$.

Appendix III. Derivation of the PGF $\Gamma(s|b)$ and Its First Derivative

In this Appendix, α, β and ψ defined in Appendix II are used again. Based on Equation (17) we can write

$$\Gamma(s|b) = \mathrm{E}\left\{ s^{R_1}\ \mathrm{E}\left[\mathrm{E}\left[s^{L_1} \middle| \underline{Z}_1 \right] \middle| R_1, |B_1'(R_1)| \right] \middle| |B_1| = b \right\} =$$

$$= \sum_{r=0}^{\infty} s^r \sum_{j=0}^{m-b} \begin{pmatrix} m-b \\ j \end{pmatrix} \sum_{z=0}^{1} \mathrm{E}\left[s^{L_1} \middle| \underline{Z}_1 = z \right] \mathrm{P}\left(\underline{Z}_1 = z \middle| |B_1'(r)| = j, |B_1| = b \right) \mathrm{P}\left(R_k = r, |B_1'(r)| = j \middle| |B_1| = b \right) =$$

$$= \sum_{r=0}^{\infty} s^r \sum_{j=0}^{m-b} \begin{pmatrix} m-b \\ j \end{pmatrix} \left[s^{\ell}\alpha + s^{\ell'}(1-(1-p)^{b+j}-\alpha) \right] (1-\lambda)^{r(m-b-j)} (1-p)^{rb} \times$$

$$\times \left[\sum_{w=1}^{r} (1-\lambda)^{w-1} \lambda (1-p)^{r-w} \right]^j . \qquad \text{(A7)}$$

Evaluating the summation over w and then the infinite series over r, we obtain

$$\Gamma(s|b) = \sum_{j=0}^{m-b} \begin{pmatrix} m-b \\ j \end{pmatrix} \left(\frac{\lambda}{\lambda-p} \right)^j \left[s^{\ell}\alpha + s^{\ell'}(1-(1-p)^{b+j}-\alpha) \right] \left\{ \sum_{n=0}^{j} \binom{j}{n} (-1)^{j-n} / (1-s(1-\beta)) \right\}, \qquad \text{(A8)}$$

$$\Gamma^{(1)}(1|b) = \sum_{j=0}^{m-b} \begin{pmatrix} m-b \\ j \end{pmatrix} \left(\frac{\lambda}{\lambda-p} \right)^j \Bigg(\psi\left[\ell\alpha + \ell'(1-(1-p)^{b+j}-\alpha) \right] + (1-(1-p)^{b+j}) \times$$

$$\times \left\{ \sum_{n=0}^{j} \binom{j}{n} (-1)^{j-n} (1-\beta)/\beta^2 \right\} \Bigg) . \qquad \text{(A9)}$$

Table 1. Steady State Measures for Case 1 where m=50, ℓ=75, ℓ'=3, λ=1.6E-4

($p_L(1)$=1.6344E-2, $p_U(50)$=8.4378E-2)

p	B^+	$E(\lvert B\rvert)$	$V^{(1)}(1\vert\vartheta)$	$F^{(1)}(1\vert\vartheta)$	T	S	D
1.144E-2	1						∞^*
1.308E-2	1	2.2886	-.1067	3.9976	98.32	427.39	491.15
1.634E-2	-	2.0754	-.1370	2.9448	91.44	296.99	357.26
1.961E-2	-	1.9466	-.1598	2.4314	86.31	233.08	291.19
2.288E-2	-	1.8611	-.1773	2.1347	82.41	195.91	252.57
2.768E-2	-	1.7778	-.1964	1.8762	78.13	163.28	218.49
3.902E-2	-	1.6721	-.2242	1.5871	71.82	126.27	179.65
5.036E-2	-	1.6194	-.2398	1.4565	68.11	109.17	161.73
6.170E-2	-	1.5887	-.2494	1.3832	65.63	99.31	151.51
7.594E-2	-	1.5655	-.2570	1.3273	63.43	91.55	143.64
8.438E-2	-	1.5563	-.2601	1.3047	62.42	88.29	140.43
1.097E-1	39 - 50	1.5405	-.2655	1.2611	60.14	81.63	134.19
1.434E-1	29 - 50	1.5337	-.2679	1.2312	58.00	76.36	129.89
1.603E-1	26 - 50	1.5336	-.2679	1.2223	57.14	74.49	128.62
1.727E-1	25 - 50						∞^*

* The system is unstable with $V^{(1)}(1\vert\vartheta) \geq 0$.

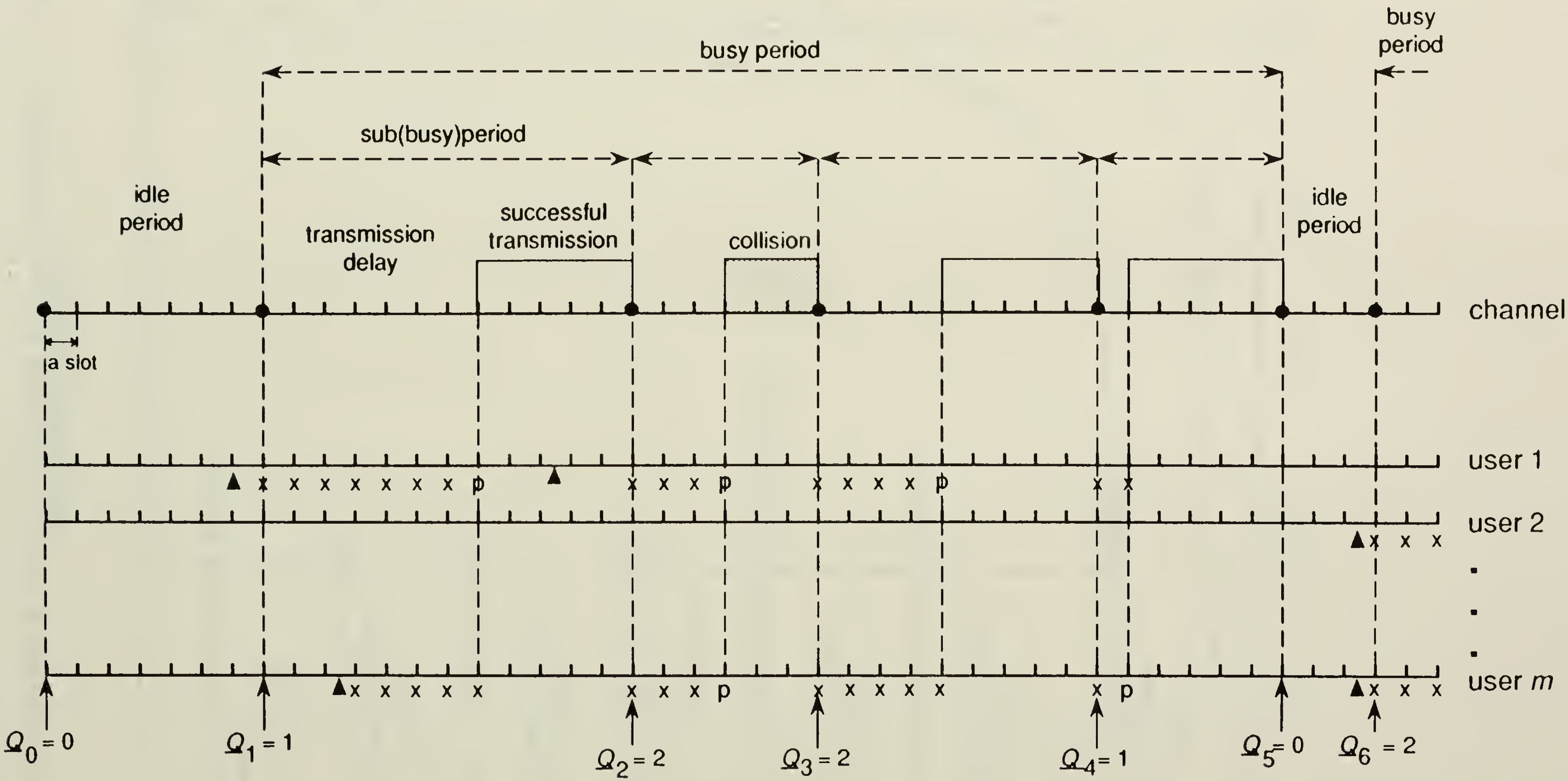
Table 2. Steady State Measures for Case 2 where m=50, ℓ=25, ℓ'=3, λ=1.2E-4

($p_L(1)$=4.8303E-2, $p_U(50)$=1.0722E-1)

p	B^+	E($\|B\|$)	$V^{(1)}(1\|\theta)$	$F^{(1)}(1\|\theta)$	T	S	D
2.415E-3	1,2						∞^*
2.898E-3	1	2.4953	-.0841	6.5053	139.23	964.57	1065.35
3.381E-3	1	2.1872	-.1200	4.1352	128.33	577.65	670.06
4.347E-3	1	1.8287	-.1843	2.3321	109.55	284.79	366.04
4.830E-3	-	1.7190	-.2111	1.9372	101.95	220.65	297.99
7.245E-3	-	1.4373	-.3040	1.1881	76.62	97.77	163.21
2.190E-2	-	1.1502	-.4556	.7534	37.61	22.98	70.03
3.896E-2	-	1.0991	-.4920	.7062	28.19	14.25	56.52
5.361E-2	-	1.0831	-.5042	.6928	24.75	11.66	52.25
7.309E-2	-	1.0733	-.5119	.6846	22.24	9.97	49.45
1.072E-1	-	1.0677	-.5163	.6791	19.96	8.59	47.32
1.608E-1	33 - 50	1.0691	-.5152	.6775	18.21	7.66	46.33
1.823E-1	29 - 50	1.0712	-.5135	.6778	17.77	7.46	46.27
2.037E-1	26 - 50	1.0738	-.5115	.6784	17.39	7.30	46.28
2.144E-1	24 - 50						∞^*

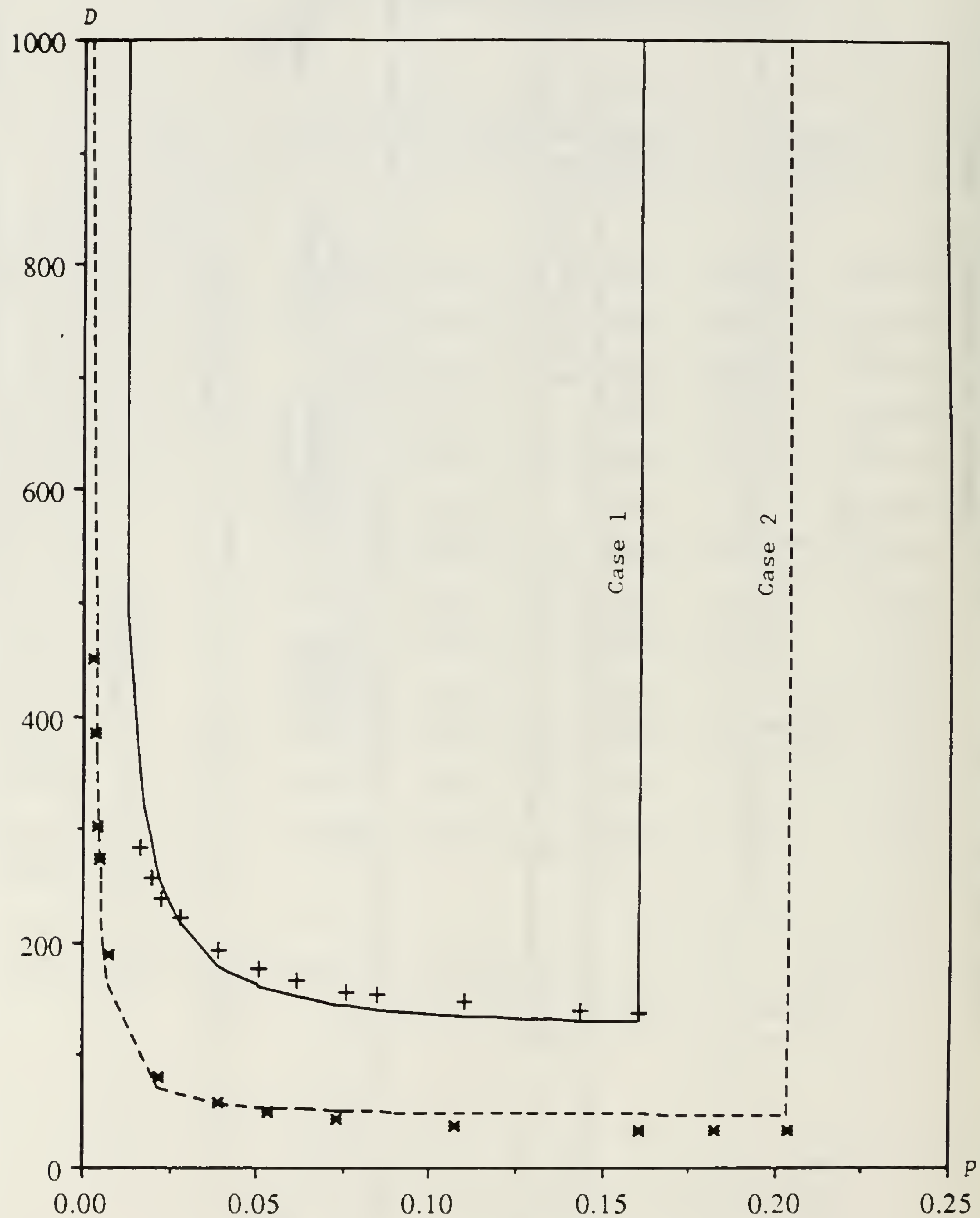
* The system is unstable with $V^{(1)}(1|\theta) \geq 0$.

Figure 1. Channel States and Queueing Process in a Slotted p-Persistent CSMA/CD System



Legend: ▲ generation of a new packet
x transmission attempt sampling a random number greater than or equal to p
p transmission attempt sampling a random number smaller than p
Q_k total packet backlog in the system at the *kth* embedded Makov epoch

Figure 2. Comparison between Approximation and Simulation: D versus p



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